

Statistical behaviour of the interference produced by MF TDMA VSAT networks

Américo A. Rubin de Celis Vidal^{a1} and José Mauro Fortes^b

^aCenter for Telecommunication Studies, PUC-Rio

Received on September 25th, 2016 / accepted on October 10th, 2016

Abstract

This paper presents a probabilistic model that can be used to assess the statistical behavior of the interference produced by the uplinks of a MF-TDMA VSAT network into a victim link of a neighbouring satellite network. In the model, analytical expressions were developed to account for the effects of the transmitting powers, antenna sizes, and transmitting antenna pointing errors. Only the earth station locations are considered via Monte Carlo simulation. Numerical results obtained with the proposed model are compared to those based on the actual values of earth station locations, antenna sizes and transmitting powers, which were provided by a Brazilian satellite operator.

Keywords: MF-TDMA VSAT networks, interference.

1. Introduction

The Multi-Frequency Time-Division Multiple Access (MF-TDMA) protocol can be used in geostationary satellite networks to provide high data rate services. It is commonly used, for example, in the inbound links of VSAT (Very Small Aperture Terminals) satellite networks. Under this protocol, a given frequency channel is shared, in time, by the transmissions of earth stations located at different sites, possibly using different transmitting antenna sizes, having different pointing errors and operating with different transmitting powers. As a consequence, the interfering power produced by the uplinks of a MF-TDMA VSAT network into a victim link of an adjacent satellite has time variations that need to be considered by any interference analysis involving such systems. This scenario is illustrated in Figure 1. In this respect, studies and analyses performed within the Radiocommunication Sector of the International Telecommunication Union (ITU-R) in the last five years led to Recommendation ITU-R S.2029 [1]. In a recent study [2] the sensitivity of the produced interference to the locations of the interfering

¹E-mail Corresponding Author: arubin@cetuc.puc-rio.br

earth stations was investigated. In all these studies, the effect of the parameters involved (e.g. earth station location, transmitting powers, pointing errors) was evaluated via Monte Carlo (MC) simulation. Depending on the size of the scenario being analyzed the computational cost associated with MC methods can be high. In Section 2, a mathematical model to characterize the statistical behavior of the interference produced by the uplinks of a MF-TDMA VSAT network is developed. In the model, analytical expressions were obtained to account for the effects of the transmitting powers, antenna sizes, and transmitting antenna pointing errors. Only the earth station locations are considered via MC simulation. In Section 3, numerical results obtained with the proposed model are compared to those based on the actual values of earth station locations, antenna sizes and transmitting powers, which were provided by a Brazilian satellite operator.

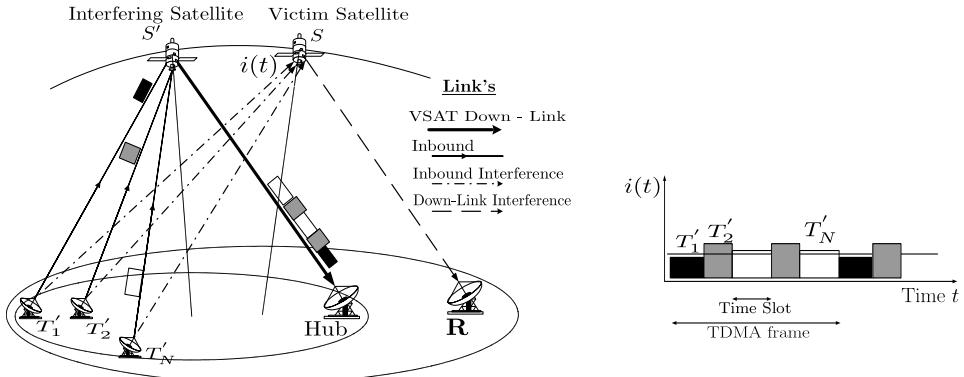


Figure 1: Interference produced by the uplinks of a MF-TDMA VSAT network into a victim link of an adjacent satellite.

2. Mathematical Model

The geometry used to calculate the interference produced by the inbound links of a VSAT network (terminal to HUB station) into the link of a neighbouring satellite is illustrated in Figure 2. In the figure, a VSAT terminal T' , located at a position \mathbf{r} in the receiving beam coverage area of satellite S' , is connected to a receiving HUB station R' . A victim neighbouring satellite link (from a transmitting earth station T to a receiving earth station R , through satellite S) is also shown in the figure. Note that the victim network receives an uplink interference from the transmitting terminal T' and a down-link interference from satellite S' . Both interference components are received by the victim receiving earth station R . Based on this geometry,

the uplink interference power density affecting the victim earth station can be written as

$$i_u(\mathbf{r}) = \frac{p'_{1r}g'_{1r}(\theta_r)g_2(\rho_r)}{l''_{ur}} \gamma \quad (1)$$

where, p'_{1r} is the transmitting power density of the interfering terminal T' (in [W/Hz]), $g'_{1r}(\theta_r)$ is the T' terminal transmitting antenna gain in the direction of the victim satellite S , $g_2(\rho_r)$ is the victim satellite receiving antenna gain in the direction of the interfering terminal T' , l''_{ur} is the free space loss experienced in the path between T' and S and γ is the transmission gain from the output of victim satellite receiving antenna to output of victim earth station receiving antenna (respectively points A and B in Figure 2). Analogously, the down-link interference power density affecting the victim

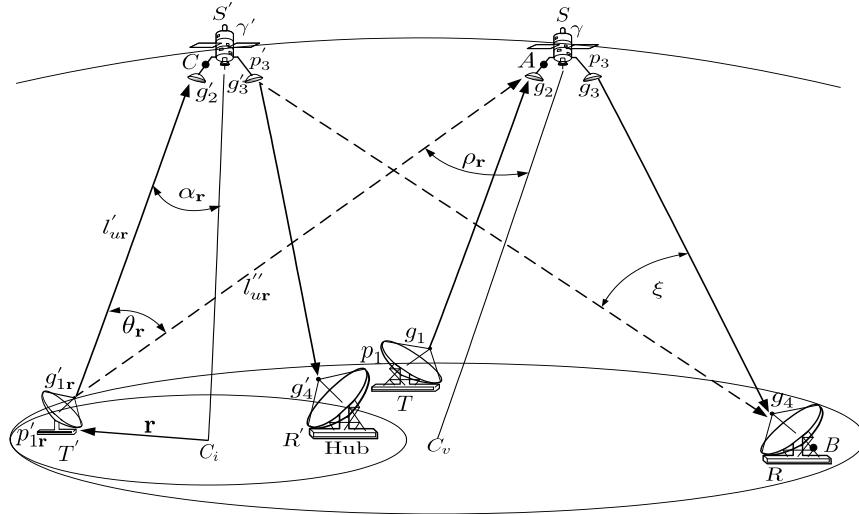


Figure 2: Interference geometry.

earth station R can be written as

$$i_d(\mathbf{r}) = \frac{p'_{1r}g'_{1r}(0)g'_2(\alpha_r)}{l'_{ur}} \gamma' \quad (2)$$

where p'_{1r} is the transmitting power density of the interfering terminal T' (in [W/Hz]), $g'_{1r}(0)$ is the T' terminal maximum transmitting antenna gain $g'_2(\rho_r)$ is the satellite S' receiving antenna gain in the direction of the interfering terminal T' , l'_{ur} is the free space loss associated with the path between T' and S' and γ' is the transmission gain from the output of satellite S' receiving antenna to output of victim earth station receiving antenna (respectively points C and B in Figure 2). Note in (1) and (2), that the index

\mathbf{r} indicates the dependence of some of the parameters with the geographical position of the earth station T' .

Note that the interference power densities in (1) and (2) can be also written in terms of the on-axis *effective isotropically radiated power (e.i.r.p.)* density radiated by the interfering terminal T' , defined as

$$e_{\mathbf{r}} = p'_{1\mathbf{r}} g'_{1\mathbf{r}}(0) \quad (3)$$

Specifically,

$$i_u(\mathbf{r}) = \frac{e_{\mathbf{r}} g'_{1\mathbf{r}}(\theta_{\mathbf{r}}) g_2(\rho_{\mathbf{r}})}{g'_{1\mathbf{r}}(0) l''_{ur}} \gamma \quad (4)$$

and

$$i_d(\mathbf{r}) = \frac{e_{\mathbf{r}} g'_2(\alpha_{\mathbf{r}})}{l'_{ur}} \gamma' \quad (5)$$

The total interference power density at the receiving earth station R is then

$$i = i_u(\mathbf{r}) + i_d(\mathbf{r}). \quad (6)$$

with $i_u(\mathbf{r})$ and $i_d(\mathbf{r})$ respectively given by (4) and (5).

Observe station R of the victim system depends of the position \mathbf{r} , the density *e.i.r.p.* $e_{\mathbf{r}}$ and the off-axis angle $\theta_{\mathbf{r}}$ associated with terminal T' . In our probabilistic approach, these quantities are modeled as random variables (the randomness of $\theta_{\mathbf{r}}$ is due to transmitting antenna pointing error). As a consequence, the interfering power i is also a random variable and its probability density function can be written as

$$p_i(I) = \int_{\Omega_{\mathbf{r}}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_{ire_{\mathbf{r}}\theta_{\mathbf{r}}}(I, \mathbf{R}, E, \Theta) dE d\Theta d\mathbf{R} \quad (7)$$

where $p_{ire_{\mathbf{r}}\theta_{\mathbf{r}}}(I, \mathbf{R}, E, \Theta)$ is the joint probability density function of the random variables i , \mathbf{r} , $e_{\mathbf{r}}$, and $\theta_{\mathbf{r}}$, that can be written as

$$p_{ire_{\mathbf{r}}\theta_{\mathbf{r}}}(I, \mathbf{R}, E, \Theta) = p_{i|\mathbf{r}=\mathbf{R}, e_{\mathbf{r}}=E, \theta_{\mathbf{r}}=\Theta}(I) p_{re_{\mathbf{r}}\theta_{\mathbf{r}}}(\mathbf{R}, E, \Theta) \quad (8)$$

where $p_{re_{\mathbf{r}}\theta_{\mathbf{r}}}(\mathbf{R}, E, \Theta)$ denote the joint probability density function of \mathbf{r} , $e_{\mathbf{r}}$, and $\theta_{\mathbf{r}}$, which can be written as

$$p_{re_{\mathbf{r}}\theta_{\mathbf{r}}}(\mathbf{R}, E, \Theta) = p_{e_{\mathbf{r}}\theta_{\mathbf{r}}|\mathbf{r}=\mathbf{R}}(E, \Theta) p_{\mathbf{r}}(\mathbf{R}), \quad (9)$$

Considering that, for a given position \mathbf{r} , the density *e.i.r.p.* density $e_{\mathbf{r}}$ and the off-axis angle $\theta_{\mathbf{r}}$ are statistically independent, we have

$$p_{e_{\mathbf{r}}\theta_{\mathbf{r}}|\mathbf{r}=\mathbf{R}}(E, \Theta) = p_{e_{\mathbf{r}}|\mathbf{r}=\mathbf{R}}(E) p_{\theta_{\mathbf{r}}|\mathbf{r}=\mathbf{R}}(\Theta) \quad (10)$$

Substituting (10), (9), (8) in (7), we get

$$p_i(I) = \int_{\Omega_r} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_{i|r=\mathbf{R}, e_r=E, \theta_r=\Theta}(I) p_{e_r|r=\mathbf{R}}(E) p_{\theta_r|r=\mathbf{R}}(\Theta) p_r(\mathbf{R}) dEd\Theta d\mathbf{R} \quad (11)$$

Note that, given $e_r = E$, $\mathbf{r} = \mathbf{R}$ e $\theta_r = \Theta$, the random variable i takes a fixed and known value $\mathcal{I}(E, \mathbf{R}, \Theta)$ with probability equal to one, meaning that

$$p_{i|r=\mathbf{R}, e_r=E, \theta_r=\Theta}(I) = \delta(I - \mathcal{I}(E, \mathbf{R}, \Theta)) \quad (12)$$

with $\delta(\cdot)$ denoting the *Dirac delta function* (impulse function). As a consequence,

$$p_i(I) = \int_{\Omega_r} p_r(\mathbf{R}) \int_{-\infty}^{\infty} p_{\theta_r|r=\mathbf{R}}(\Theta) \int_{-\infty}^{\infty} \delta(I - \mathcal{I}(E, \mathbf{R}, \Theta)) p_{e_r|r=\mathbf{R}}(E) dEd\Theta d\mathbf{R} \quad (13)$$

It is reasonable to assume that, given $\mathbf{r} = \mathbf{R}$, the *e.i.r.p.* e_r is a discrete random variable taking values in a known set of $N_{\mathbf{R}}$ values $\{E_{j_{\mathbf{R}}}, j = 1, \dots, N_{\mathbf{R}}\}$, related to the set of applicable antenna sizes and to the geographical position \mathbf{R} of the interfering terminal T' . In this case,

$$p_{e_r|r=\mathbf{R}}(E) = \sum_{j=1}^{N_{\mathbf{R}}} P_{j_{\mathbf{R}}} \delta(E - E_{j_{\mathbf{R}}}) \quad (14)$$

with

$$P_{j_{\mathbf{R}}} = P(e = E_{j_{\mathbf{R}}} \mid \mathbf{r} = \mathbf{R}) \quad (15)$$

and the inner integral in (13) becomes

$$\int_{-\infty}^{\infty} \delta(I - \mathcal{I}(E, \mathbf{R}, \Theta)) p_{e_r|r=\mathbf{R}}(E) dE = \sum_{j=1}^{N_{\mathbf{R}}} P_{j_{\mathbf{R}}} \delta(I - \mathcal{I}(E_{j_{\mathbf{R}}}, \mathbf{R}, \Theta)) \quad (16)$$

and, as a consequence, (13) is rewritten as

$$p_i(I) = \int_{\Omega_r} p_r(\mathbf{R}) \sum_{j=1}^{N_{\mathbf{R}}} P_{j_{\mathbf{R}}} \int_{-\infty}^{\infty} p_{\theta_r|r=\mathbf{R}}(\Theta) \delta(I - \mathcal{I}(E_{j_{\mathbf{R}}}, \mathbf{R}, \Theta)) d\Theta d\mathbf{R} \quad (17)$$

The probability distribution function of the interfering power density, defined as $P(i \leq I)$, is obtained by integrating (17), resulting

$$F_i(I) = \int_{\Omega_r} p_r(\mathbf{R}) \sum_{j=1}^{N_{\mathbf{R}}} P_{j_{\mathbf{R}}} \int_{-\infty}^{\infty} p_{\theta_r|r=\mathbf{R}}(\Theta) u(I - \mathcal{I}(E_{j_{\mathbf{R}}}, \mathbf{R}, \Theta)) d\Theta d\mathbf{R} \quad (18)$$

with $u(\cdot)$ denoting the unit step function. Note that (18) can be also written as

$$F_i(I) = \int_{\Omega_r} p_r(\mathbf{R}) \sum_{j=1}^{N_R} P_{jR} H_{Rj}(I) d\mathbf{R} \quad (19)$$

with

$$H_{Rj}(I) = \int_{-\infty}^{\infty} p_{\theta_r|\mathbf{r}=\mathbf{R}}(\Theta) u(I - \mathcal{I}(E_{jR}, \mathbf{R}, \Theta)) d\Theta \quad (20)$$

To obtain an analytical expression for $H_{Rj}(I)$, first note that, considering (4) (5) and (6), $\mathcal{I}(E_{jR}, \mathbf{R}, \Theta)$ can be written as

$$\mathcal{I}(E_{jR}, \mathbf{R}, \Theta) = K_{1Rj} g'_{1Rj}(\Theta) + K_{2Rj} \quad (21)$$

with

$$K_{1Rj} = \frac{E_{jR} g_2(\rho_R)}{g'_{1Rj}(0) l''_{uR}} \gamma \quad (22)$$

and

$$K_{2Rj} = \frac{E_{jR} g'_2(\alpha_R)}{l'_{uR}} \gamma' \quad (23)$$

In (21) and (22), $g'_{1Rj}(\cdot)$ represents the radiation pattern of the interfering terminal T' transmitting antenna associated with the *e.i.r.p.* level E_{jR} , when the terminal is located at $\mathbf{r} = \mathbf{R}$. This radiation pattern usually takes the form

$$g'_{1Rj}(\Theta) = \begin{cases} f_{1Rj}(\Theta) & ; \quad 0 \leq \Theta < \Theta_{1Rj} \\ G_{1Rj} & ; \quad \Theta_{1Rj} \leq \Theta < \Theta_{2Rj} \\ f_{2Rj}(\Theta) & ; \quad \Theta_{2Rj} \leq \Theta < \Theta_{3Rj} \\ G_{2Rj} & ; \quad \Theta_{3Rj} \leq \Theta \end{cases} \quad (24)$$

where the functions $f_{1Rj}(\Theta)$ and $f_{2Rj}(\Theta)$ describe the antenna main lobe and antenna sidelobes, respectively and with G_{1Rj} and G_{2Rj} denoting, respectively, the antenna near and far side-lobe levels. Note that the on-axis antenna gain G_{0Rj} is equal to $f_{1Rj}(0)$.

Considering (20) and (21), it is possible to verify that $H_{Rj}(I)$ can be written as

$$H_{Rj}(I) = \int_{S_{Rj}} p_{\theta_r|\mathbf{r}=\mathbf{R}}(\Theta) d\Theta \quad (25)$$

with

$$S_{Rj} = \left\{ \Theta \in \Omega_\theta \mid g'_{1Rj}(\Theta) \leq \frac{I - K_{1Rj}}{K_{2Rj}} \right\} \quad (26)$$

Note that the random variable θ_r depends on the azimuth and elevation pointing errors, ϕ_a and ϕ_e , of the interfering terminal transmitting antenna.

As in [2], these pointing errors are here modeled as zero mean, statistically independent gaussian random variables, with variance σ^2 . In [2], the effect of these pointing errors on θ_r and, as a consequence, on $H_{Rj}(I)$ is evaluated via Monte Carlo simulation. Here, analytical expressions are obtained for $p_{\theta_r|\mathbf{r}=\mathbf{R}}(\Theta)$ and $H_{Rj}(I)$. From the geometry in Figure 3, it can be easily seen that

$$\theta_r \approx \sqrt{(\phi_a - \phi_{a_R})^2 + (\phi_e - \phi_{e_R})^2} \quad (27)$$

with ϕ_{a_R} and ϕ_{e_R} denoting, respectively, the azimuth and elevation angles of the interfering terminal transmitting antenna when it is pointing at the victim satellite S . From (27) and considering that ϕ_a and ϕ_e are zero mean, statistically independent gaussian random variables with variance σ^2 , it is possible to show that, given $\mathbf{r} = \mathbf{R}$, θ_r has a Rician probability density function, that is,

$$p_{\theta_r|\mathbf{r}=\mathbf{R}}(\Theta) = \frac{\Theta}{\sigma^2} \exp\left(-\frac{\Theta^2 + A_{\mathbf{R}}^2}{2\sigma^2}\right) I_0\left(\frac{\Theta A_{\mathbf{R}}}{\sigma^2}\right) u(\Theta) \quad (28)$$

where $A_{\mathbf{R}} = \sqrt{\phi_{a_R}^2 + \phi_{e_R}^2}$.

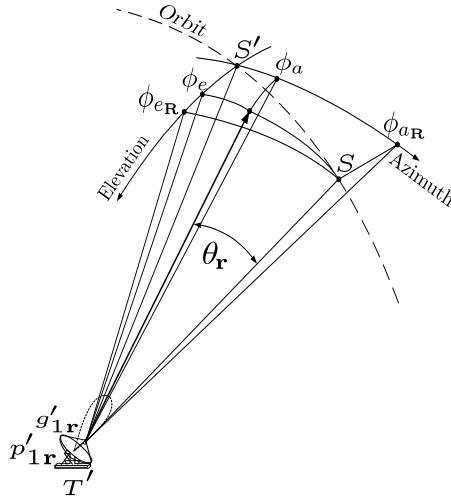


Figure 3: Dependence of θ_r on the azimuth and elevation antenna pointing errors.

In determining an analytical expression for $H_{Rj}(I)$, (25) was used with (28) as integrand. The antenna radiation pattern in (24) was used to determine the integration region \mathcal{S}_{Rj} . After some mathematical manipulations,

we get

$$H_{\mathbf{R}j}(I) = \begin{cases} 0 & ; I < G_{2\mathbf{R}j}K_{1\mathbf{R}j} + K_{2\mathbf{R}j} \\ Q_1\left(\frac{A_{\mathbf{R}}}{\sigma}, \frac{f_{2\mathbf{R}j}^{-1}\left(\frac{I-K_{2\mathbf{R}j}}{K_{1\mathbf{R}j}}\right)}{\sigma}\right) & ; G_{2\mathbf{R}j}K_{1\mathbf{R}j} + K_{2\mathbf{R}j} \leq I < G_{1\mathbf{R}j}K_{1\mathbf{R}j} + K_{2\mathbf{R}j} \\ Q_1\left(\frac{A_{\mathbf{R}}}{\sigma}, \frac{f_{1\mathbf{R}j}^{-1}\left(\frac{I-K_{2\mathbf{R}j}}{K_{1\mathbf{R}j}}\right)}{\sigma}\right) & ; G_{1\mathbf{R}j}K_{1\mathbf{R}j} + K_{2\mathbf{R}j} \leq I < G_{0\mathbf{R}j}K_{1\mathbf{R}j} + K_{2\mathbf{R}j} \\ 1 & ; G_{0\mathbf{R}j}K_{1\mathbf{R}j} + K_{2\mathbf{R}j} \leq I \end{cases} \quad (29)$$

with $Q_1(\cdot, \cdot)$ denoting the *Generalized Marcum-Q function* Q_1 .

Finally, the interference probability distribution function is determined using (19) and (29).

3. Numerical Results

In this section, (19) and (29) are used to determine the probability distribution function of the interference produced by a MF-TDMA VSAT system, operating in the Ku Band, into a link of a neighbouring satellite. The geographical locations \mathbf{r} of the VSAT terminals is modeled by a two-dimensional homogeneous Poisson Point Process (PPP) [3] with density λ terminals per km^2 .

Two examples, involving different densities of terminals are presented. In examples a high gain receiving antenna for the victim earth station R is assumed. In this case, it is possible to consider that the gain $g_4(0)$ in the direction of the victim satellite S is much larger than the gain $g_4(\xi)$ in the direction of the interfering network satellite S' , that is $g_4(0) \gg g_4(\xi)$, being then reasonable to assume that the gain γ associated with the satellite S is much larger than γ' associated with the satellite S' , that is $\gamma \gg \gamma'$. Under these hypotheses the interfering power density $i_u(\mathbf{r})$ is dominant, allowing us to ignore the effects of interference $i_d(\mathbf{r})$, that is, $i(\mathbf{r}) \approx i_u(\mathbf{r})$. The antenna radiation pattern in (24) was assumed to be that in Recommendation ITU-R F.1245. The interfering and the victim satellites were placed, respectively, at longitudes 65°W and 62°W . The victim satellite receiving antenna is assumed to have maximum gain at $(54.9^\circ\text{W}, 11.7^\circ\text{S})$ and a radiation pattern as that in Recommendation ITU-R S.672 with $\psi_0 = 3^\circ$, $a = 3.16$, $b = 6.32$ and $L_s = -30 \text{ dBi}$). The azimuth and elevation pointing errors standard deviations were taken to be 0.447° .

In Example 1 the VSAT terminals are distributed within a $750,000 \text{ km}^2$ region in the eastern part of the Brazilian coast with $\lambda = 2.4 \times 10^{-5}$ terminals per km^2 . Only 0.96 m diameter antennas were considered for the terminals

($N_{\mathbf{R}} = 1, P_{j_{\mathbf{R}}} = 1$ for all \mathbf{R}). The *e.i.r.p.* density $E_{j_{\mathbf{R}}}$ was determined as the maximum *e.i.r.p.* density value such that the off-axis *e.i.r.p.* density limits in Recommendation ITU-R S.524 are satisfied, being equal to -3.42 dB(W/Hz).

In Example 2 the VSAT terminals are distributed within a 5,100,000 km² region in the northeast cost of Brazil with $\lambda = 5.1 \times 10^{-6}$ terminals per km². Antennas with diameters 0.96 m and 1.8 m were considered for the terminals ($N_{\mathbf{R}} = 2, P_{1_{\mathbf{R}}} = 0.1, P_{1_{\mathbf{R}}} = 0.9$). The *e.i.r.p.* densities were again determined as the maximum *e.i.r.p.* density values such that the off-axis *e.i.r.p.* density limits in Recommendation ITU-R S.524 are satisfied, resulting $E_{1_{\mathbf{R}}} = -3.42$ dB(W/Hz) and $E_{2_{\mathbf{R}}} = 3.29$ dB(W/Hz).

In both examples, the interference was normalized with respect to a reference value i_{ref} , that is, the statistical behaviour of the random variable $i_n = i/i_{ref}$ was determined. This reference value was determined as the interference power density affecting the victim link when the VSAT interfering terminal is placed at the geographical location corresponding to the maximum gain of the victim satellite receiving antenna and the pointing errors ϕ_a and ϕ_e are equal to zero. The resulting complementary probability density functions corresponding to examples 1 and 2 are presented in Figure 4. For comparison purposes, Figure 4 includes results produced with actual terminal locations, antenna sizes and *e.i.r.p* density values, for four different sets of values of the VSAT terminals antenna pointing errors (randomly chosen).

Note that there is a reasonable agreement (approximately 1 dB difference) between the results obtained with the proposed model and those based on actual data. The differences are larger in Example 2 than in Example 1. This is expected since, in comparison with Example 1, Example 2 has a lower density of terminals and a larger number of possible antenna sizes.

4. Conclusion

This paper has presented a probabilistic model to assess the statistical behavior of the interference produced by the uplinks of a MF-TDMA VSAT network into a victim link of a neighbouring satellite network. Analytical expressions were developed to account for the effects of the VSAT transmitting powers, antenna sizes, and transmitting antenna pointing errors. Earth station locations were modeled as a two dimensional homogeneous Poisson Point Process. The application of the model to two example scenarios has indicated a good agreement between the results produced with the proposed model and those based on actual network data. The next step would be to

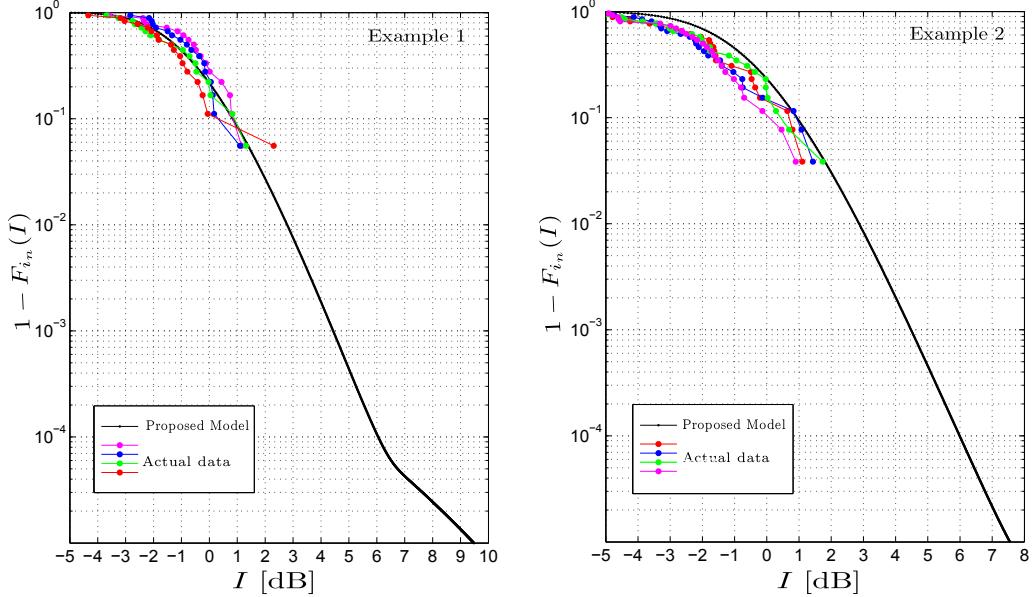


Figure 4: Complementary probability density functions corresponding to examples 1 and 2.

modify the proposed model to make it applicable to large regions containing multiple subregions with different density of terminals and propagation characteristics.

References

- [1] *Statistical methodology to access time-varying interference produced by a geostationary fixed-satellite service network of earth stations operating with MF-TDMA schemes to geostationary fixed-satellite networks*, Recommendation ITU-R S.2029, International Telecommunication Union, Geneva, 2012.
- [2] *Methodology to estimate the sensitivity of GSO FSS interference levels to the geographical location of earth stations communicating with satellites in the fixed-satellite service in the 14 GHz and 29.5-30 GHz frequency bands*, Annex 21, Doc. 4A/591, International Telecommunication Union, Geneva, July 2014.
- [3] Chiu S., Stoyan D., Kendall W. and Mecke J., *Stochastic Geometry and its Applications*, Third Edition, John Wiley & Sons, 2013.